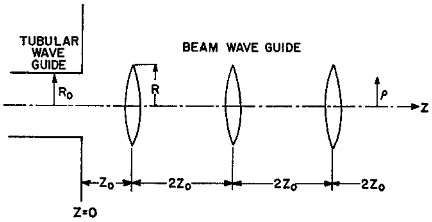

Fig. 1. The launching structure.

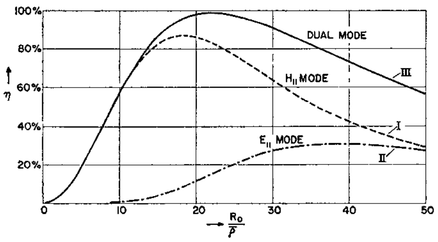

Fig. 2. The launching efficiency η of the dominant beam mode as a function of R_0/ρ_0.

———————— Excitation by a superposition of a H_{11}-mode and an E_{11}-mode (with optimum amplitude ratio of the two modes).
– – – – – – – Excitation by a H_{11}-mode
—·—·—·— Excitation by an E_{11}-mode.

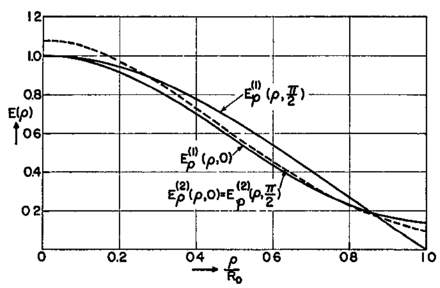

Fig. 3. The cross-sectional electric field distribution in the waveguide aperture in the case of optimum launching conditions for the dominant beam mode. The dotted curves shows the Gaussian field distribution of the beam mode.

Inserting (1) and (4) into (9) the integrals over ρ and ϕ can be evaluated; expressing furthermore C by (7) the power ratio η becomes

$$\eta = \frac{\left\{F\left(\bar{a}_{11}, \frac{R_0}{\rho_0}\right) + \frac{B}{A} F\left(a_{11}, \frac{R_0}{\rho_0}\right)\right\}^2}{\alpha^2 + \left(\frac{B}{A}\right)^2 \cdot \beta^2} \tag{10}$$

where

$$\alpha^2 = \tfrac{1}{2}(\bar{a}_{11}^2 - 1)\bar{a}_{11}^{-2} J_0^2(\bar{a}_{11}) = 0.40458 \qquad \beta^2 = \tfrac{1}{2} a_{11}^2 J_0^2(a_{11}) = 1.1908.$$

Equation (10) yields the launching efficiency as a function of R_0/ρ_0 and of the amplitude ratio B/A between the E_{11}- and H_{11}-modes. A simple calculation shows that η as a function of B/A has a maximum for

$$\frac{B}{A} = \frac{\alpha^2}{\beta^2} \cdot \frac{F\left(a_{11}, \frac{R_0}{\rho_0}\right)}{F\left(\bar{a}_{11}, \frac{R_0}{\rho_0}\right)}. \tag{11}$$

The maximum value itself is with (10)

$$\hat{\eta} = \left[\frac{F\left(\bar{a}_{11}, \frac{R_0}{\rho_0}\right)}{\alpha}\right]^2 + \left[\frac{F\left(a_{11}, \frac{R_0}{\rho_0}\right)}{\beta}\right]^2. \tag{12}$$

The integrals F (8) on the right-hand side of this equation have been evaluated by a computer as functions of R_0/ρ_0. With the numerical values obtained $\hat{\eta}$ according to (12) has been plotted in Fig. 2. The optimum launching efficiency is reached at $R_0/\rho_0 = 2.15$; the optimum value, as stated before, is 98.3 percent. Note that for a confocal beam waveguide ($z_0 = f$), the optimum aperture radius of the metallic waveguide, according to (5), is $R_0 = 3.04\sqrt{z_0/k}$.

The dotted curves in Fig. 2 show the launching efficiency if the dominant beam mode is excited by only the H_{11}-mode ($B=0$) or by only the E_{11}-mode ($A=0$). In the first case, as stated before, a launching efficiency of 86.7 percent can be achieved; in the second case the launching efficiency does not exceed 31 percent.

In Fig. 3 the tangential electric field distribution in the waveguide aperture is plotted for optimum launching conditions; for $R_0/\rho_0 = 2.15$ the amplitude ratio B/A according to (11) is

$$\frac{B}{A} = 0.250. \tag{13}$$

(The corresponding power ratio of the E_{11}-mode and the H_{11}-mode is 0.184.) The curves show the components $E_\rho^{(1)}$ and $E_\phi^{(1)}$ as functions of ρ for $\phi = 0$ and $\phi = \pi/2$, respectively; these components have been normalized to unity at $\rho = 0$. For a comparison the Gaussian distribution of the components $E_\rho^{(2)}$, $E_\phi^{(2)}$ of the dominant beam mode in the plane $z=0$ is indicated by the dotted line; their amplitude factor C ($=1.08$) is with (7), determined by the normalization of $E_\rho^{(1)}$, $E_\phi^{(1)}$.

Potter[5] has treated the radiation from the open end of a cylindrical waveguide illuminated by a superposition of a H_{11}-mode and an E_{11}-mode. If the amplitude ratio of the two modes is in the neighborhood of the value (13) a radiation pattern with a low sidelobe level is obtained. The minimum possible backlobe is achieved for an amplitude ratio that differs from the value (13) by less than 2 percent; in this case the sidelobe level is below 44 dB. These results are in agreement with our calculations as the radiation characteristic of a Gaussian wave beam has no sidelobes. Sidelobes must be accounted for by the higher-order beam modes which, in the case of optimum launching efficiency for the dominant beam mode, are excited only with very small amplitudes.

ACKNOWLEDGMENT

The authors are indebted to Dr. G. Goubau for his interest in this work, and to Mrs. H. Perlman of the Mathematics Analysis Section, U. S. Army Electronics Command, for performing the computer evaluation.

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Comments on Loaded Q of a Waveguide Cavity Resonator

Several equations relating the doubly loaded Q of a lossless waveguide cavity resonator to the normalized susceptance of the input/output inductive coupling obstacles (i.e., posts or irises) have appeared in the literature. These equations employ transcendental functions that are not convenient for many design calculations. In this correspondence, an approximate equation will be presented that does not employ transcendental functions and gives reasonably accurate results for most practical cases.

A single cavity resonator symmetrically loaded by matched waveguide terminations through inductive discontinuities can be represented by the equivalent circuit shown in Fig. 1. For a nominal half-wave resonator, resonance occurs when

$$\tan\phi = \frac{2}{b} \tag{1}$$

where

b = normalized susceptance of discontinuity (Note: b will be *negative* for *inductive* irises or posts)
$\phi = 2\pi l/\lambda_{g0}$ = cavity electrical length
l = cavity physical length
λ_{g0} = guide wavelength at resonance.

Assuming b varies linearly with λ_g (i.e., $B = K\lambda_g$ where K is a constant), the loaded Q of the resonant cavity has been derived by Reed [1]

$$Q_L = \frac{1}{4}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \left[-b\sqrt{b^2+4} \cdot \tan^{-1}\left(\frac{2}{b}\right) + \frac{2b^2}{\sqrt{b^2+4}}\right] \tag{2}$$

where

Q_L = loaded Q of resonant cavity
λ_0 = free-space wavelength at resonance.

Manuscript received August 30, 1966; revised November 2, 1966. This work was performed while the author was with the RCA Advanced Communications Lab., New York, N. Y.

This exact expression for Q_L takes into account the frequency sensitivity of b. It should be noted that the center frequency is neither the geometric nor algebraic mean of the half-power frequencies [2].

Approximate equations for loaded Q are available that neglect the frequency sensitivity of b. In Mumford's classic paper on waveguide filters [3], the equation for loaded Q is

$$Q_L = \frac{1}{4}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \cdot \left[-b\sqrt{b^2+4}\tan^{-1}\left(\frac{2}{b}\right)\right]. \quad (3)$$

Riblet [4] employs an expression for Q_L as

$$Q_L = \frac{(\pi-\phi)\cos\phi}{\sin^2\phi}. \quad (4)$$

Letting $\phi=\tan^{-1}(1/b)$, (4) becomes identical with (3).

Pritchard [5] and Fano and Lawson [6] compute the loaded Q from the following as

$$Q_L = \left(\frac{1+b^2}{4}\right)\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \cdot \tan^{-1}\left(\frac{2b}{b^2-1}\right). \quad (5)$$

This equation—which has also appeared in Ragan (vol. 9 of the Radiation Laboratory Series) [7]—which presents the following simplified equation applicable to narrow-band filters when $b \gg 10$:

$$Q_L = \frac{\pi b^2}{4}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2. \quad (6)$$

An approximate equation for loaded Q which does not employ transcedental functions will now be derived [8].

The lumped-circuit prototype of the doubly-loaded lossless waveguide cavity resonator is shown in Fig. 2. All circuit parameters have been normalized. Replacing the shunt loadings by equivalent series loadings, Fig. 3 is now applicable.

$$Q_L = \frac{\omega_0 l + 2x}{2r} = \frac{\omega_0 l'}{2r}. \quad (7)$$

For a short-circuited $\lambda_g/2$ transmission line, $\omega_0 l + 2x$ is, to a good approximation, equal to

$$\frac{\omega_0}{2Z_0}\left.\frac{dX}{d\omega_0}\right|_{\omega_0} = \frac{\omega_0 L}{Z_0} = \omega_0 l' \quad (8)$$

where

ω_0 = angular resonant frequency
X = absolute reactance of waveguide transmission line
Z_0 = waveguide characteristic impedance.

Since $X=Z_0\tan 2\pi l/\lambda_g$, then [9]

$$\frac{\omega_0}{2Z_0}\left.\frac{dX}{d\omega}\right|_{\omega_0} = \frac{\pi}{2}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 = \omega_0 l'. \quad (9)$$

To obtain r as shown in Fig. 3

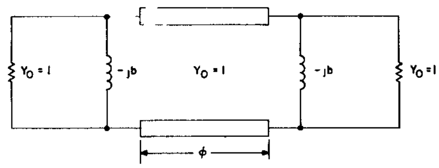

Fig. 1. Equivalent circuit of waveguide cavity resonator.

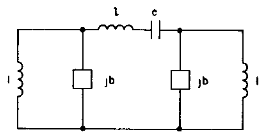

Fig. 2. Lumped-circuit prototype with shunt loadings.

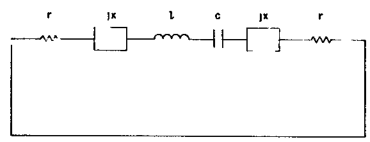

Fig. 3. Lumped-circuit prototype with series loadings.

let

$$z = r + jx = \frac{1}{1+jb} = \frac{1}{1+b^2} - \frac{jb}{1+b^2} \quad (10)$$

then

$$r = \frac{1}{1+b^2}. \quad (11)$$

Substituting (9) and (11) into (7), it can be shown that

$$Q_L = \frac{\pi}{4}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2(1+b^2). \quad (12)$$

Values of loaded Q for different normalized susceptances have been calculated using (2), (3), (5), (6), and (12). These are tabulated below, assuming $(\lambda_{g0}/\lambda_0)^2 = 2.08$:

	Q_L				
B	equation (2)	equation (3)	equation (5)	equation (6)	equation (12)
2	8.44	6.93	5.79	6.56	8.16
4	28.8	24.9	22.9	26.2	27.8
6	61.8	55.7	54.5	59.0	60.4
8	108	99.8	98.1	105	106.5
10	166	156.5	154	164	165
12	238	226	226	236	237

It can be seen that the approximate equation (12) provides the closest results when compared to exact equation (2). Errors of less than 3.3 percent will be incurred for values of $b \geqq 2.0$. When $b \geqq 10.0$, this error is reduced to a maximum of 0.6 percent. It should be noted that (2), (3), (5), (6), and (12) are applicable to the singly-loaded waveguide cavity by replacing the factor $\frac{1}{4}$ by $\frac{1}{2}$.

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Control of Resonant Frequency of YIG-Disk Filter by Doublet Tuning

I. Introduction

Magnetically tunable filters generally use ferrimagnetic resonators in the shape of a sphere. The most common ferrimagnetic material is single-crystal yttrium iron garnet (YIG), which has a saturation magnetization $4\pi M_s$ of approximately 1750 Oe. YIG spheres are satisfactory at frequencies down to almost 2 GHz, but doping is required (for instance, with gallium) to obtain high-Q resonance at lower frequencies. Since it is difficult to control the $4\pi M_s$ of Ga YIG, tuning adjustments have to be provided in multi-resonator filters, usually by a rotation of one of the Ga YIG spheres on a dielectric rod, changing the angles between its crystalline axes and the applied magnetic field.[1] The lower the operating frequency of the filter, the more doping is required, and the greater the variation in $4\pi M_s$ from resonator to resonator. By using disks rather than spheres, less doping is required.[2] However, it is more difficult to adjust the resonant frequency of a disk by rotation, than it is for a sphere. Another technique was therefore developed, using a pair of disks with adjustable spacing between them for each resonator; the spacing is adjusted to control the resonant frequency of the composite resonator.

In order to realize a multi-resonator band-pass filter with one Ga YIG disk per resonator, one must find a set of disks whose resonant frequencies at a given magnetic field, including the effects of shape and saturation, are sufficiently matched.[2] For example, in ordering disks (e.g., from Airtron, with nominal $4\pi M_s = 1000$ Oe), a tolerance less than ±10

Manuscript received September 12, 1966. This work was supported by the U. S. Army Electronics Laboratories, Fort Monmouth, N. J. under Contract DA28-043 AMC-01371.

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